# Differential distributions at NNLO

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# NNLO computations are in a (pre-) revolutionary phase:

- new promising techniques are in use for two-loop integrals,
- NLO wisdom (and tools) start being applied at NNLO,
- generic subtraction approaches for double real radiation reach maturity

| Process | State of the Art | Desired |
|---|---|---|
| H | $d\sigma$ @ NNLO QCD (expansion in $1/m_t$)<br>full $m_t/m_b$ dependence @ NLO QCD<br>and @ NLO EW<br>NNLO+PS, in the $m_t \to \infty$ limit | $d\sigma$ @ NNNLO QCD (infinite-$m_t$ limit)<br>full $m_t/m_b$ dependence @ NNLO QCD<br>and @ NNLO QCD+EW<br>NNLO+PS with finite top quark mass effects |
| H + j | $d\sigma$ @ NNLO QCD (g only)<br>and finite-quark-mass effects<br>@ LO QCD and LO EW | $d\sigma$ @ NNLO QCD (infinite-$m_t$ limit)<br>and finite-quark-mass effects<br>@ NLO QCD and NLO EW |
| H + 2j | $\sigma_{tot}$(VBF) @ NNLO(DIS) QCD<br>$d\sigma$(VBF) @ NLO EW<br>$d\sigma$(gg) @ NLO QCD (infinite-$m_t$ limit)<br>and finite-quark-mass effects @ LO QCD | $d\sigma$(VBF) @ NNLO QCD + NLO EW<br><br>$d\sigma$(gg) @ NNLO QCD (infinite-$m_t$ limit)<br>and finite-quark-mass effects<br>@ NLO QCD and NLO EW |
| H + V | $d\sigma$ @ NNLO QCD<br>$d\sigma$ @ NLO EW<br>$\sigma_{tot}$(gg) @ NLO QCD (infinite-$m_t$ limit) | with H $\to$ bb @ same accuracy<br>$d\sigma$(gg) @ NLO QCD<br>with full $m_t/m_b$ dependence |
| tH and $\bar{t}$H | $d\sigma$(stable top) @ LO QCD | $d\sigma$(top decays)<br>@ NLO QCD and NLO EW |
| $t\bar{t}$H | $d\sigma$(stable tops) @ NLO QCD | $d\sigma$(top decays)<br>@ NLO QCD and NLO EW |
| gg $\to$ HH | $d\sigma$ @ NLO QCD (leading $m_t$ dependence)<br>$d\sigma$ @ NNLO QCD (infinite-$m_t$ limit) | $d\sigma$ @ NLO QCD<br>with full $m_t/m_b$ dependence |

**Table 1:** Wishlist part 1 – Higgs (V = W, Z)

| Process | State of the Art | Desired |
|---|---|---|
| V | $d\sigma$(lept. V decay) @ NNLO QCD<br>$d\sigma$(lept. V decay) @ NLO EW | $d\sigma$(lept. V decay) @ NNNLO QCD<br>and @ NNLO QCD+EW<br>NNLO+PS |
| V + j(j) | $d\sigma$(lept. V decay) @ NLO QCD<br>$d\sigma$(lept. V decay) @ NLO EW | $d\sigma$(lept. V decay)<br>@ NNLO QCD + NLO EW |
| VV′ | $d\sigma$(V decays) @ NLO QCD<br>$d\sigma$(on-shell V decays) @ NLO EW | $d\sigma$(decaying off-shell V)<br>@ NNLO QCD + NLO EW |
| gg $\to$ VV | $d\sigma$(V decays) @ LO QCD | $d\sigma$(V decays) @ NLO QCD |
| V$\gamma$ | $d\sigma$(V decay) @ NLO QCD<br>$d\sigma$(PA, V decay) @ NLO EW | $d\sigma$(V decay)<br>@ NNLO QCD + NLO EW |
| V$b\bar{b}$ | $d\sigma$(lept. V decay) @ NLO QCD<br>massive b | $d\sigma$(lept. V decay) @ NNLO QCD<br>+ NLO EW, massless b |
| VV′$\gamma$ | $d\sigma$(V decays) @ NLO QCD | $d\sigma$(V decays)<br>@ NLO QCD + NLO EW |
| VV′V″ | $d\sigma$(V decays) @ NLO QCD | $d\sigma$(V decays)<br>@ NLO QCD + NLO EW |
| VV′ + j | $d\sigma$(V decays) @ NLO QCD | $d\sigma$(V decays)<br>@ NLO QCD + NLO EW |
| VV′ + jj | $d\sigma$(V decays) @ NLO QCD | $d\sigma$(V decays)<br>@ NLO QCD + NLO EW |
| $\gamma\gamma$ | $d\sigma$ @ NNLO QCD + NLO EW | $q_T$ resummation at NNLL matched to NNLO |

# As a result the Les Houches wishlist was recently radicalized!

As the LHC enters its precision era improving our understanding of all processes that are related to Higgs signal and backgrounds becomes a priority. All these processes involve colorless final states.

The need emerges to integrate NNLO corrections in one code, including decays and sometimes QCD/EW corrections to decays- e.g.

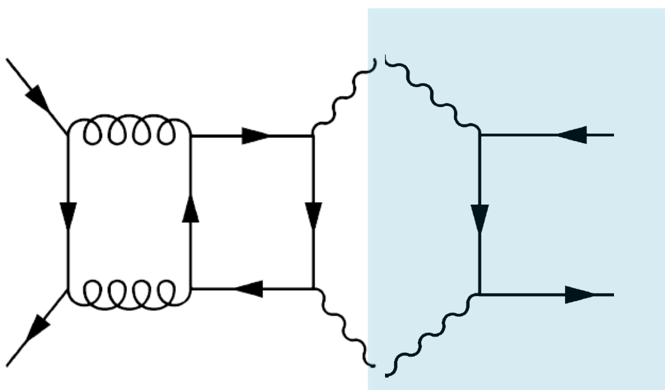
$$pp \to VH \to Vb\bar{b}$$

see Grazzini, Ferera, Tramontano arXiv:1312.1669

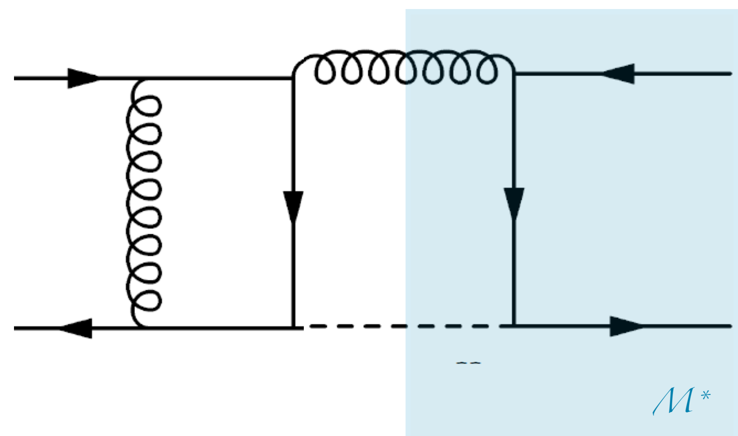
Our ~~short~~ medium-term goal is to provide NNLO differential distributions for all Higgs-related colorless final states including decays, in a unified framework.

First step: a parallelized code for Higgs production in gluon fusion, see talk by Franz Herzog

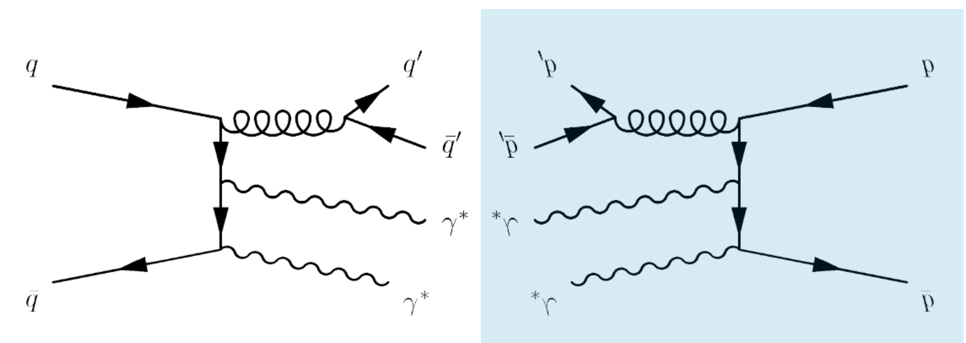
# Challenges in nnlo $pp \rightarrow$colorless computations



2-loop Virtual




Real-Virtual

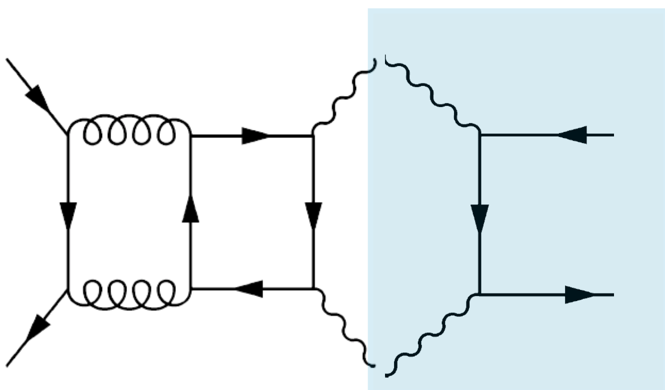



Double Real

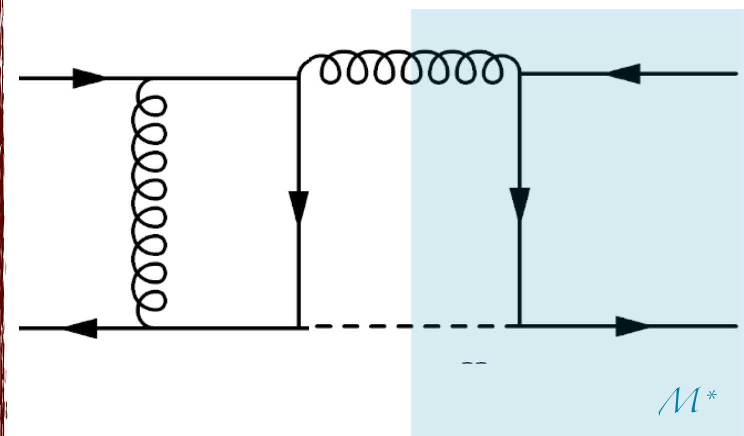
## Usually the show-stopper, because of lack of master integrals.

However there is much progress in analytic tools for master integrals recently, e.g. Caola, Melnikov, Henn, Smirnov 1404.5590, 1402.7078 , Gehrmann, Manteuffel, Tancredi, Weihs 1404.4853, 1306.6344, Duhr, Chavez 1209.2722

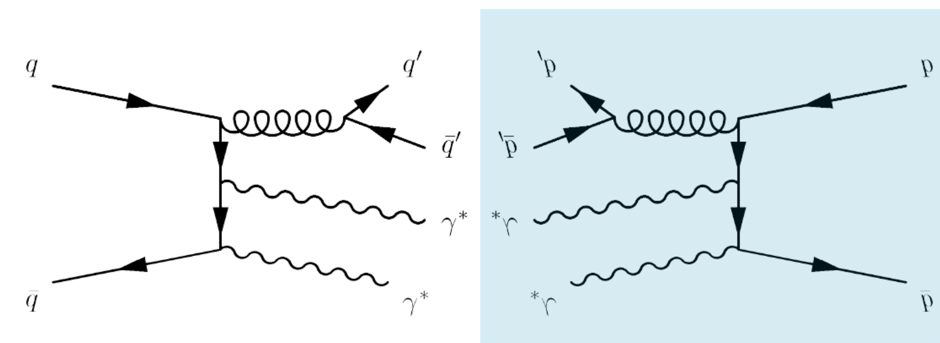
# Challenges in nnlo $pp \to$colorless computations



2-loop Virtual



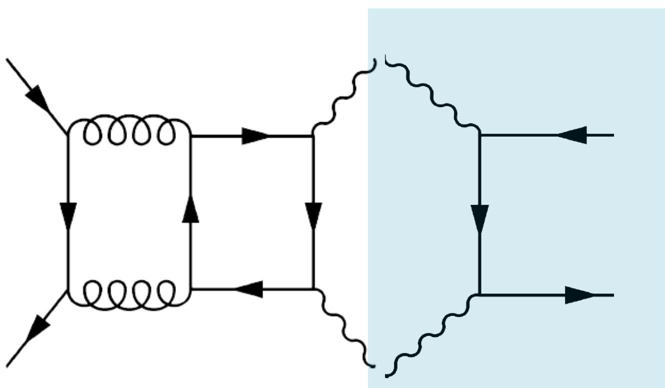
Real-Virtual



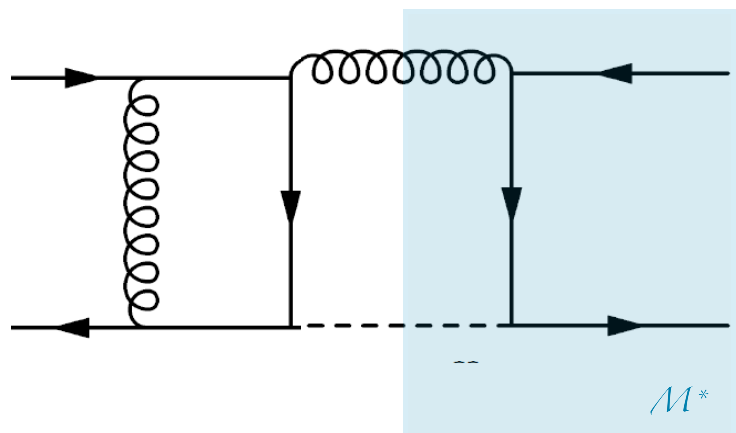
Double Real

The soft and collinear limits necessary for subtraction are known or easily derivable. However stable one loop amplitudes as these limits are approached are still a major issue. Which of the one-loop providers can we use and to which extent?

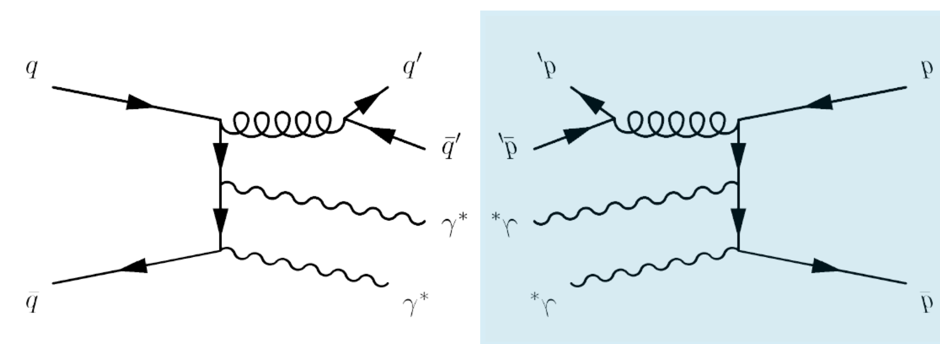
# Challenges in nnlo $pp \to$colorless computations



2-loop Virtual



Real-Virtual



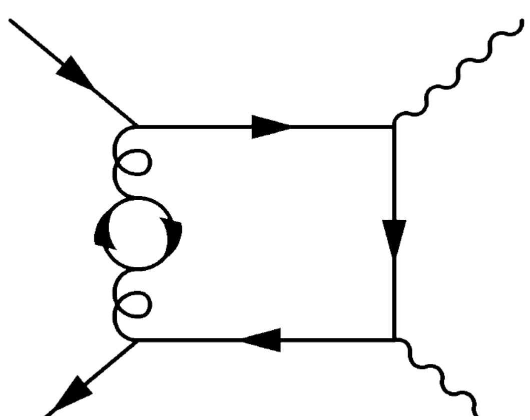
Double Real

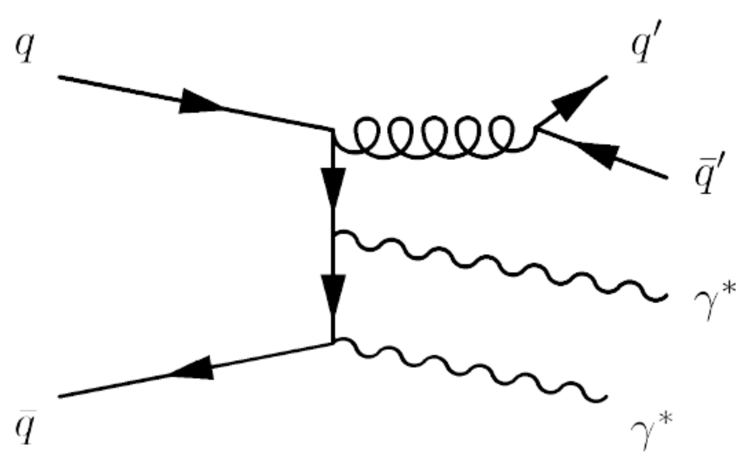
In principle the double real subtraction problem is solved by all the methods in the market: Qt subtraction, sector decomposition with topologies, topologies with non-linear mappings, phase-space selectors plus sector decomposition, antennas...

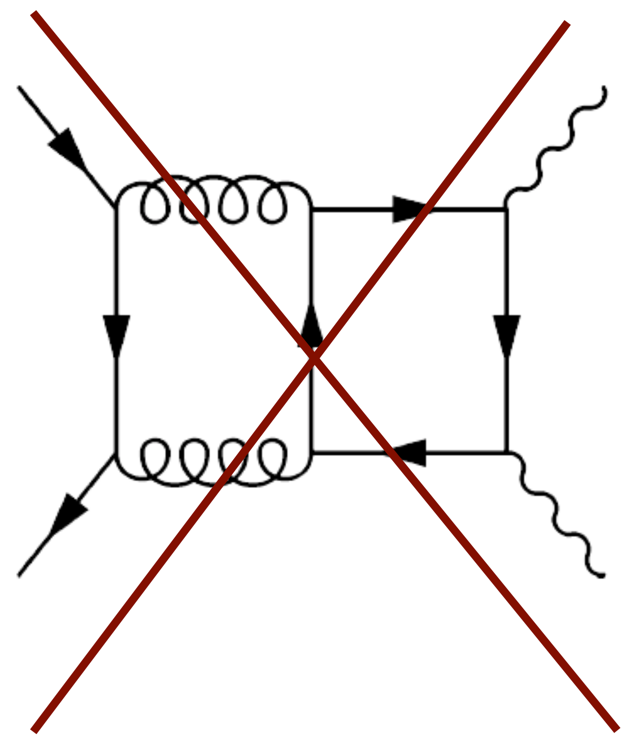
# the most trivial example with two final state particles: the nf pieces of $pp \to \gamma^*\gamma^*$

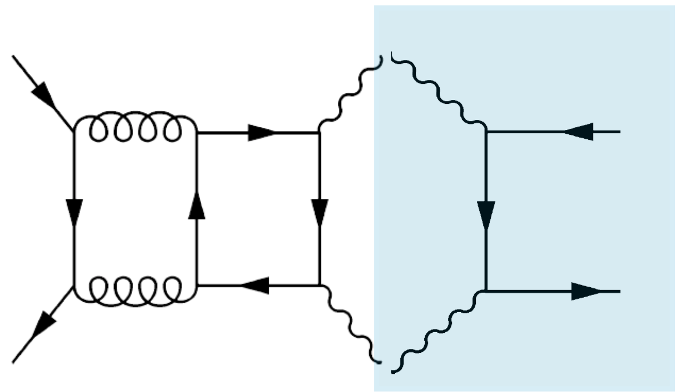
**[ with R. Mueller, F. Chavez, C. Duhr, B. Anastasiou, J. Cancino ]**

- No real-virtual
- Challenging but not monumental double virtual
- Easiest possible double real





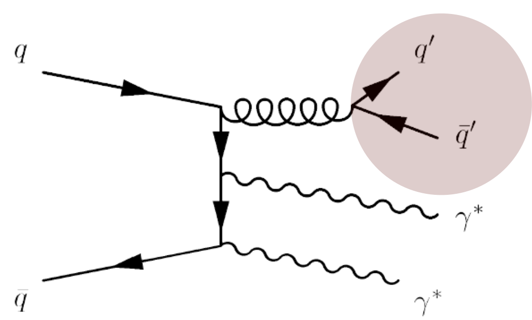





The two-loop virtual diagrams were reduced to master integrals with IBP reduction (using AIR). The master integrals were done in the spirit of Duhr, Chavez 1209.2722. They are a subset of those published by Caola, Melnikov, Henn, Smirnov 1404.5590, 1402.7078. The techniques used deserve a whole talk.

Instead, I will focus on the treatment of the double real.

# integrating out the final state quark pair




$$d\Phi_{12\to q\bar{q}\gamma^*\gamma^*} = \frac{s_{12}\ dz}{2\pi}\frac{ds_g}{2\pi} d\Phi_{12\to g^* Q} d\Phi_{g^*\to q\bar{q}} d\Phi_{Q\to\gamma^*\gamma^*}$$

$$\int d\Phi_{g^*\to q\bar{q}} |M_{12\to q\bar{q}\gamma^*\gamma^*}|^2 = \frac{A(\epsilon)}{s_g^{1+\epsilon}} |M_{12\to g^*\gamma^*\gamma^*}|^2$$

$$A(\epsilon) = 2g_s^2 N_F \frac{d-2}{d-1}\frac{1}{2}\frac{\Omega_{d-1}}{(4\pi)^{d-2}}$$

$$p_g = \bar{z}\bar{\lambda}p_1 + \bar{z}\lambda\frac{1-\rho\bar{z}\bar{\lambda}}{1-\bar{z}\bar{\lambda}}p_2 + \bar{z}\sqrt{s_{12}\rho\lambda\bar{\lambda}}\ e_T$$

Parametrize the off-shell gluon in the hierarchical parametrization

# integrating out the final state quark pair

$$p_g = p_{q'} + p_{\bar{q}'} \parallel p_1$$

$$|M_{12\to g^*\gamma^*\gamma^*}|^2 \sim \frac{-4g_s^2}{\tilde{s}_{1g}}\frac{B_1(z)}{z}P_{qq;1}(z,\rho)$$

$$\tilde{s}_{1g} = -s_{12}\bar{z}\lambda.$$

$$P_{qq;1}(z,\rho) = C_F\left[\frac{2}{\bar{z}} - 2 + (1-\epsilon)\bar{z}\rho\right]$$

$$p_g = p_{q'} + p_{\bar{q}'} \parallel p_2$$

$$|M_{12\to g^*\gamma^*\gamma^*}|^2 \sim \frac{-4g_s^2}{\tilde{s}_{2g}}\frac{B_2(z)}{z}P_{qq;2}(z,\rho)$$

$$\tilde{s}_{2g} = -s_{12}\bar{z}\bar{\lambda}(1-\bar{\rho}\bar{z})$$

$$P_{qq;2}(z,\rho) = C_F\left[\frac{2}{\bar{z}} - 2 + (1-\epsilon)\bar{z}(1 - \frac{z\bar{\rho}}{1-\bar{z}\bar{\rho}})\right]$$

$$p_{q'} \parallel p_{\bar{q}'}$$

$$\lim_{\rho\to 1}|M_{12\to g^*\gamma^*\gamma^*}|^2 = |M_{12\to g\gamma^*\gamma^*}|^2$$

The triple collinear limits are easy to derive and they commute with the final state collinear limit.

# double real with integrated quarks

$$d\sigma_{RR}^{U} = d\sigma_{HH} + d\sigma_{R;C} + d\sigma_{CC_1} + d\sigma_{CC_2}$$

$$\begin{aligned} d\sigma_{HH} = & \frac{1}{2s_{12}} \frac{s_{12}\, dz}{2\pi} \frac{ds_g}{2\pi} \frac{A(\epsilon)}{s_g^{1+\epsilon}} \rho^{-\epsilon} d\Phi_{12\to gQ} d\Phi_{Q\to\gamma^*\gamma^*} \\ & \times \left[ |M_{12\to g^*\gamma^*\gamma^*}|^2 - \frac{4g_s^2}{-\tilde{s}_{1g}} P_{qq;1}(z,\rho) \frac{B_1(z)}{z} - \frac{4g_s^2}{-\tilde{s}_{2g}} P_{qq;2}(z,\rho) \frac{B_2(z)}{z} \right. \\ & \left. - |M_{12\to g\gamma^*\gamma^*}|^2 + \frac{4g_s^2}{-\tilde{s}_{1g}^*} P_{qq}(z) \frac{B_1(z)}{z} + \frac{4g_s^2}{-\tilde{s}_{2g}^*} P_{qq}(z) \frac{B_2(z)}{z} \right], \end{aligned}$$

The subtracted double real is automatically soft-finite (non-trivial)

# Triple collinear counter-terms integrated

$$\int_{\lambda,\rho} d\sigma_{CC_{1,2}} = \left(\frac{\alpha_s}{\pi}\right)^2 \left(\frac{\mu^2}{s_{12}}\right)^{2\epsilon} G^{NNLO}_{1,2}(z) d\sigma_{B_{1,2}}(z) dz$$

$$\begin{aligned}
G_1^{NNLO}(z) = \frac{C_F N_F}{48} \Bigg\{ & -\frac{\delta(\bar{z})}{\epsilon^3} + \frac{1}{\epsilon^2}\left[4\mathcal{D}_0(\bar{z}) - \frac{5}{3}\delta(\bar{z}) - 2(1+z)\right] \\
& + \frac{1}{\epsilon}\Bigg[-16\mathcal{D}_1(\bar{z}) + \frac{20}{3}\mathcal{D}_0(\bar{z}) - \frac{1}{18}(56 - 21\pi^2)\delta(\bar{z}) \\
& - \frac{10}{3}(1+z) + 8(1+z)\log\bar{z} + 2(1+z^2)\frac{\log z}{\bar{z}}\Bigg] \\
& + 32\mathcal{D}_2(\bar{z}) - \frac{80}{3}\mathcal{D}_1(\bar{z}) + \frac{2}{9}(56 - 21\pi^2)\mathcal{D}_0(\bar{z}) \\
& - \frac{1}{54}(328 - 105\pi^2 - 1116\zeta_3)\delta(\bar{z}) \\
& - 4(1+z^2)\frac{\mathrm{Li}_2(\bar{z})}{\bar{z}} - 16(1+z)\log^2\bar{z} - (1+z^2)\frac{\log^2 z}{\bar{z}} - 8(1+z^2)\frac{\log z \log\bar{z}}{\bar{z}} \\
& + \frac{40}{3}(1+z)\log\bar{z} + \frac{10}{3}(1+z^2)\frac{\log z}{\bar{z}} \\
& - \frac{1}{9}(38 + 74z + (1+z)(-21\pi^2))\Bigg\} + \mathcal{O}(\epsilon), \qquad (6.28)
\end{aligned}$$

$$G_2^{NNLO}(z) = G_1^{NNLO}(z) + \frac{C_F N_F}{48}\left(4(1+z^2)\frac{\mathrm{Li}_2(\bar{z})}{\bar{z}} - 4\log z - 4\bar{z}\right) + \mathcal{O}(\epsilon). \qquad (6.29)$$

The triple collinear counter-terms can be analytically integrated

# Final state collinear counter-term integrated

$$
\begin{aligned}
d\sigma_{R;C} - \frac{\alpha_s}{\pi}\frac{\beta_0|_{N_F}}{\epsilon}d\sigma_H &= d\sigma^{RC}_{NLO} + \frac{\alpha_s}{\pi}\frac{N_F}{6\epsilon}d\sigma_H \\
&= \frac{1}{2s_{12}}\frac{s_{12}dzd\lambda d\phi}{2\pi}d\Phi_{Q\to\gamma\gamma}\,\bar{z}\left(\frac{\alpha_s}{\pi}\right)^2\frac{N_F}{6}\left[-\frac{5}{3}+\log\left(\frac{s_{12}\bar{z}^2\lambda\bar{\lambda}}{\mu^2(1-\bar{z}\bar{\lambda})}\right)\right] \\
&\quad\times\left(\frac{|M_{12\to g\gamma^*\gamma^*}|^2}{4g_s^2} - \frac{P_{qq}(z)}{2z(-\tilde{s}^*_{1q})}B_1(z) - \frac{P_{qq}(z)}{2z(-\tilde{s}^*_{2q})}B_2(z)\right)
\end{aligned}
$$

The single collinear counter-term is also analytically integrated and is seen to be proportional to the NLO subtracted real emission amplitude.

# Symmetrizing the parametrization

$$p_g = \bar{t}\left[\bar{\lambda} p_1 + \lambda p_2 + \sqrt{s_{12}\lambda\bar{\lambda}\rho}\, e_T\right]$$

$$\bar{t} \equiv \frac{1-\sqrt{1-4\lambda\bar{\lambda}\bar{z}\bar{\rho}}}{2\lambda\bar{\lambda}\bar{\rho}}$$

$$|M_{12\to g^*\gamma\gamma}|^2 \sim \frac{-4g_s^2}{\tilde{s}_{1g}}\frac{P_{qq;S}(z,\rho)}{2}\frac{B_1(z)}{z} + \mathcal{O}(\lambda^0), \qquad \text{as} \quad \lambda \to 0,$$

$$|M_{12\to g^*\gamma\gamma}|^2 \sim \frac{-4g_s^2}{\tilde{s}_{2g}}\frac{P_{qq;S}(z,\rho)}{2}\frac{B_2(z)}{z} + \mathcal{O}(\bar{\lambda}^0), \qquad \text{as} \quad \lambda \to 1,$$

$$P_{qq;S}(z,\rho) = C_F\frac{1+z^2-\epsilon\bar{z}^2-\bar{\rho}\bar{z}(1+z-\epsilon\bar{z})}{\bar{z}(1-\bar{\rho}\bar{z})}$$

$$G_S(z) \equiv G_1(z) - \frac{C_F N_F}{48}(4\log z + 4\bar{z})$$

Alternative parametrization with symmetric counter-terms

# Subtraction fully exclusively

$$p_{q'} = \bar{z}\Bigg[\bar{x}_1 x_3\, p_1 + x_1\left(x_3\bar{x}_2 + \frac{x_2\bar{x}_3}{z+\bar{z}x_1} - 2\cos\pi x_4\sqrt{\frac{x_2\bar{x}_2x_3\bar{x}_3}{z+\bar{z}x_1}}\right)p_2$$

$$- \sqrt{s_{12}\frac{x_1\bar{x}_1}{\bar{x}_2}}\left(x_3\bar{x}_2 - \cos\pi x_4\sqrt{\frac{x_2\bar{x}_2x_3\bar{x}_3}{z+\bar{z}x_1}}\right)e_T^{\mu} + \sin\pi x_4\sqrt{s_{12}\frac{x_1\bar{x}_1x_2x_3\bar{x}_3}{z+\bar{z}x_1}}\; e'_T\Bigg]$$

$$p_{\bar{q}'} = \bar{z}\Bigg[\bar{x}_1 \bar{x}_3\, p_1 + x_1\left(\bar{x}_3\bar{x}_2 + \frac{x_2 x_3}{z+\bar{z}x_1} + 2\cos\pi x_4\sqrt{\frac{x_2\bar{x}_2x_3\bar{x}_3}{z+\bar{z}x_1}}\right)p_2$$

$$- \sqrt{s_{12}\frac{x_1\bar{x}_1}{\bar{x}_2}}\left(\bar{x}_3\bar{x}_2 + \cos\pi x_4\sqrt{\frac{x_2\bar{x}_2x_3\bar{x}_3}{z+\bar{z}x_1}}\right)e_T - \sin\pi x_4\sqrt{s_{12}\frac{x_1\bar{x}_1x_2x_3\bar{x}_3}{z+\bar{z}x_1}}\; e'_T\Bigg]$$

$$d\sigma_{HH}^{excl.} = \frac{g_s^4}{8(2\pi)^4}\frac{s_{12}dz}{2\pi}dx_1\dots dx_4 d\phi\,\bar{z}$$

$$\times\Bigg[\left(\frac{\bar{z}^2x_1\bar{x}_1}{z+\bar{z}x_1}\right)\frac{|M_{12\to q'\bar{q}'\gamma^*\gamma^*}|^2}{4g_s^4} - \left(\frac{\bar{z}^2x_1}{z}\right)\frac{P_{qq;1}^{excl.}}{\tilde{s}_{1g}}\frac{B_1}{z} - \left(\bar{z}^2\bar{x}_1\right)\frac{P_{qq;2}^{excl.}}{\tilde{s}_{2g}}\frac{B_2}{z}$$

$$- \left(\frac{\bar{z}^2x_1\bar{x}_1}{z+\bar{z}x_1}\right)\frac{P_{\mu\nu}}{s_g}\frac{\mathcal{M}^{\mu\nu}_{12\to g\gamma^*\gamma^*}}{2g_s^2} + \left(\frac{\bar{z}^2x_1}{z}\right)\frac{\tilde{P}_{qq;1}^{excl.}}{\tilde{s}^*_{1g}}\frac{B_1}{z} + \left(\bar{z}^2\bar{x}_1\right)\frac{\tilde{P}_{qq;2}^{excl.}}{\tilde{s}^*_{2g}}\frac{B_2}{z}\Bigg]$$

We can also do the subtraction fully exclusively (the integrated counter-terms are the same as before).

# Subtraction fully exclusively

$$p_{q'} = \bar{z}\Bigg[\bar{x}_1 x_3\, p_1 + x_1\left(x_3\bar{x}_2 + \frac{x_2\bar{x}_3}{z+\bar{z}x_1} - 2\cos\pi x_4\sqrt{\frac{x_2\bar{x}_2x_3\bar{x}_3}{z+\bar{z}x_1}}\right)p_2$$
$$- \sqrt{s_{12}\frac{x_1\bar{x}_1}{\bar{x}_2}}\left(x_3\bar{x}_2 - \cos\pi x_4\sqrt{\frac{x_2\bar{x}_2x_3\bar{x}_3}{z+\bar{z}x_1}}\right)e_T^\mu + \sin\pi x_4\sqrt{s_{12}\frac{x_1\bar{x}_1x_2x_3\bar{x}_3}{z+\bar{z}x_1}}\;e'_T\Bigg]$$

$$p_{\bar{q}'} = \bar{z}\Bigg[\bar{x}_1\bar{x}_3\, p_1 + x_1\left(\bar{x}_3\bar{x}_2 + \frac{x_2x_3}{z+\bar{z}x_1} + 2\cos\pi x_4\sqrt{\frac{x_2\bar{x}_2x_3\bar{x}_3}{z+\bar{z}x_1}}\right)p_2$$
$$- \sqrt{s_{12}\frac{x_1\bar{x}_1}{\bar{x}_2}}\left(\bar{x}_3\bar{x}_2 + \cos\pi x_4\sqrt{\frac{x_2\bar{x}_2x_3\bar{x}_3}{z+\bar{z}x_1}}\right)e_T - \sin\pi x_4\sqrt{s_{12}\frac{x_1\bar{x}_1x_2x_3\bar{x}_3}{z+\bar{z}x_1}}\;e'_T\Bigg]$$

$$d\sigma_{HH}^{excl.} = \frac{g_s^4}{8(2\pi)^4}\frac{s_{12}dz}{2\pi}dx_1\ldots dx_4 d\phi\,\bar{z}$$
$$\times\Bigg[\left(\frac{\bar{z}^2x_1\bar{x}_1}{z+\bar{z}x_1}\right)\frac{|M_{12\to q'\bar{q}'\gamma^*\gamma^*}|^2}{4g_s^4} - \left(\frac{\bar{z}^2x_1}{z}\right)\frac{P^{excl.}_{qq;1}}{\tilde{s}_{1g}}\frac{B_1}{z} - (\bar{z}^2\bar{x}_1)\frac{P^{excl.}_{qq;2}}{\tilde{s}_{2g}}\frac{B_2}{z}$$
$$- \left(\frac{\bar{z}^2x_1\bar{x}_1}{z+\bar{z}x_1}\right)\frac{P_{\mu\nu}}{s_g}\frac{\mathcal{M}^{\mu\nu}_{12\to g\gamma^*\gamma^*}}{2g_s^2} + \left(\frac{\bar{z}^2x_1}{z}\right)\frac{\tilde{P}^{excl.}_{qq;1}}{\tilde{s}^*_{1g}}\frac{B_1}{z} + (\bar{z}^2\bar{x}_1)\frac{\tilde{P}^{excl.}_{qq;2}}{\tilde{s}^*_{2g}}\frac{B_2}{z}\Bigg]$$

We then recover the full spin correlations

# Subtraction fully exclusively

$$P^{excl.}_{qq;1} = \lim_{x_1\to 0} C_F \frac{s_{134}}{4s_{34}}\Bigg[-\frac{1}{s_{134}s_{34}}\left(\frac{2(s_{14}z_3 - s_{13}z_4)}{z_3+z_4} + \frac{s_{34}(z_3-z_4)}{z_3+z_4}\right)^2 + (1-2\epsilon)\left(-\frac{s_{34}}{s_{134}} + z_3 + z_4\right) + \frac{4z_1 + (z_3-z_4)^2}{z_3+z_4}\Bigg]$$

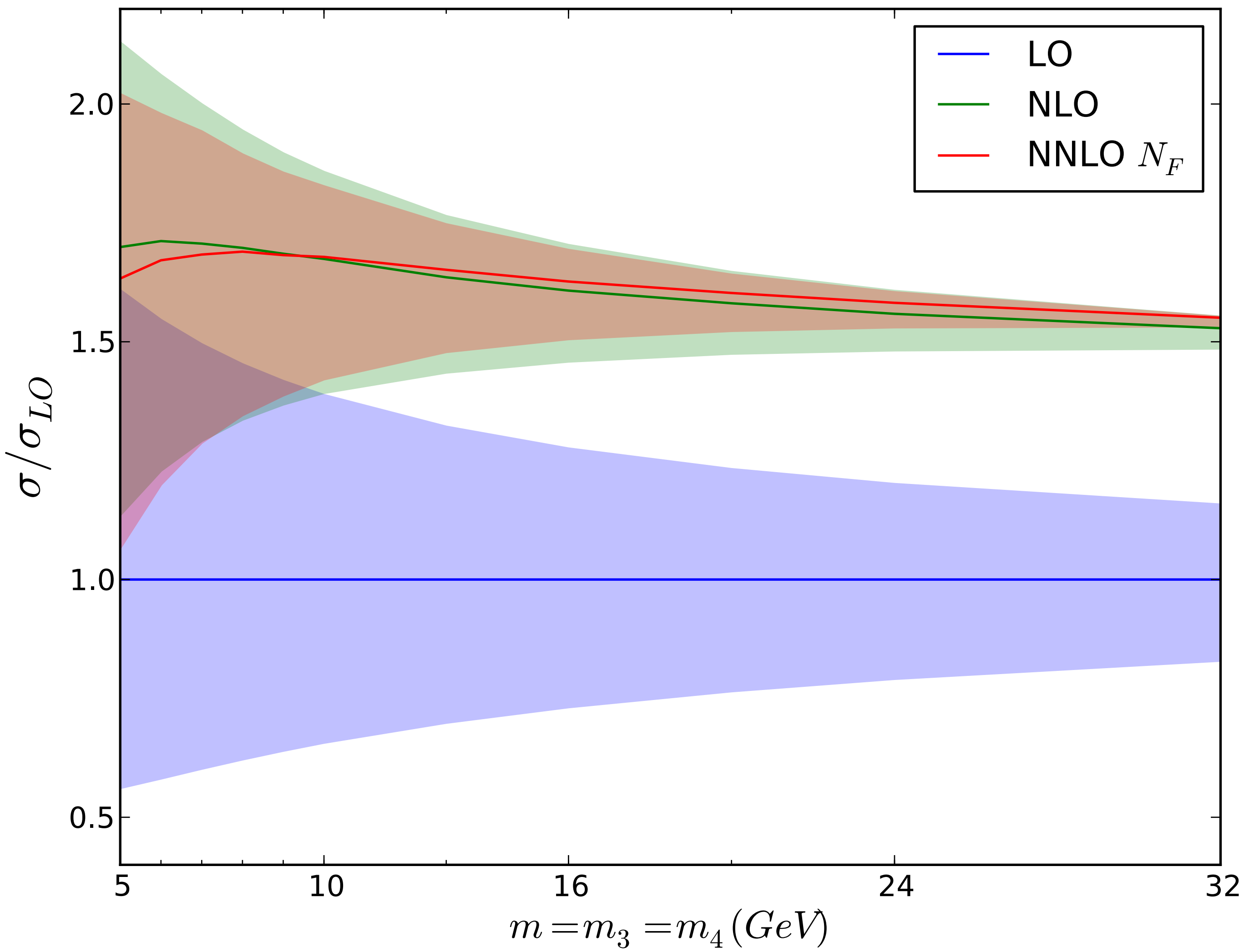
$$= \frac{C_F z}{2x_2\bar{z}^2}\Big[\left(1+z^2\right)\bar{x}_2(1-2x_3\bar{x}_3) + 8zx_2x_3\bar{x}_3 - \bar{x}_2\bar{z}^2\epsilon - 4z\bar{x}_2x_3\bar{x}_3\cos(2\pi x_4) + 4(1+z)(1-2x_3)\sqrt{zx_2\bar{x}_2x_3\bar{x}_3}\cos(\pi x_4)\Big], \tag{6.50}$$

$$P^{excl.}_{qq;2} = \lim_{x_1\to 1} C_F \frac{s_{234}}{4s_{34}}\Bigg[-\frac{1}{s_{234}s_{34}}\left(\frac{2(s_{24}z_3 - s_{23}z_4)}{z_3+z_4} + \frac{s_{34}(z_3-z_4)}{z_3+z_4}\right)^2 + (1-2\epsilon)\left(-\frac{s_{34}}{s_{234}} + z_3 + z_4\right) + \frac{4z_1 + (z_3-z_4)^2}{z_3+z_4}\Bigg]$$

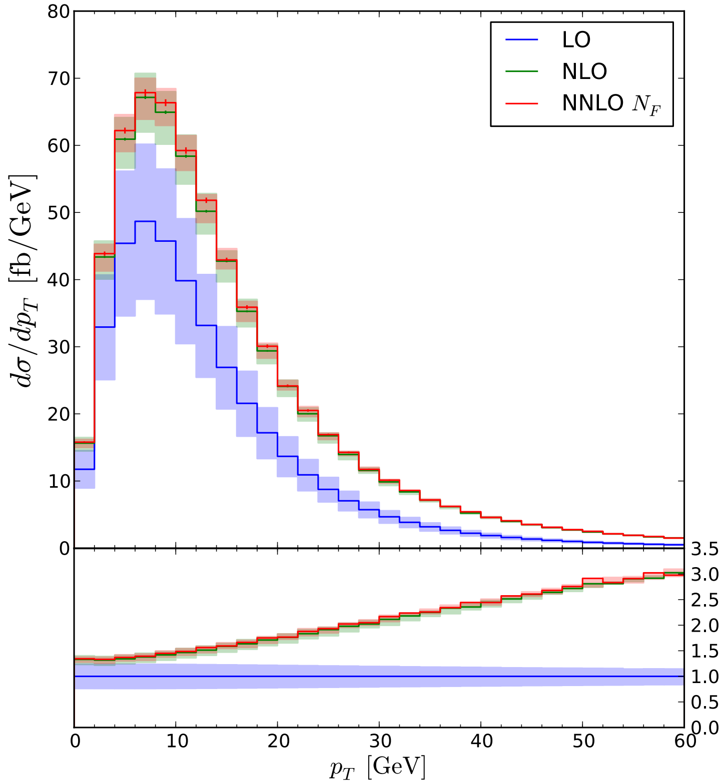
$$= \frac{C_F}{2x_2\bar{z}^2}\Big[2x_2^2\bar{z}(2-x_2\bar{z})(1-6x_3\bar{x}_3) + (1+x_2)\left(1+z^2\right)(1-2x_3\bar{x}_3) - 4x_2(1-2x_3)^2 - \epsilon\bar{x}_2\bar{z}^2 - 4(1-x_2\bar{z})(z-x_2\bar{z})\bar{x}_2x_3\bar{x}_3\cos(2\pi x_4) + 4(1+z-2x_2\bar{z})(1-2x_3)(1-x_2\bar{z})\sqrt{x_2\bar{x}_2x_3\bar{x}_3}\cos(\pi x_4)\Big], \tag{6.51}$$

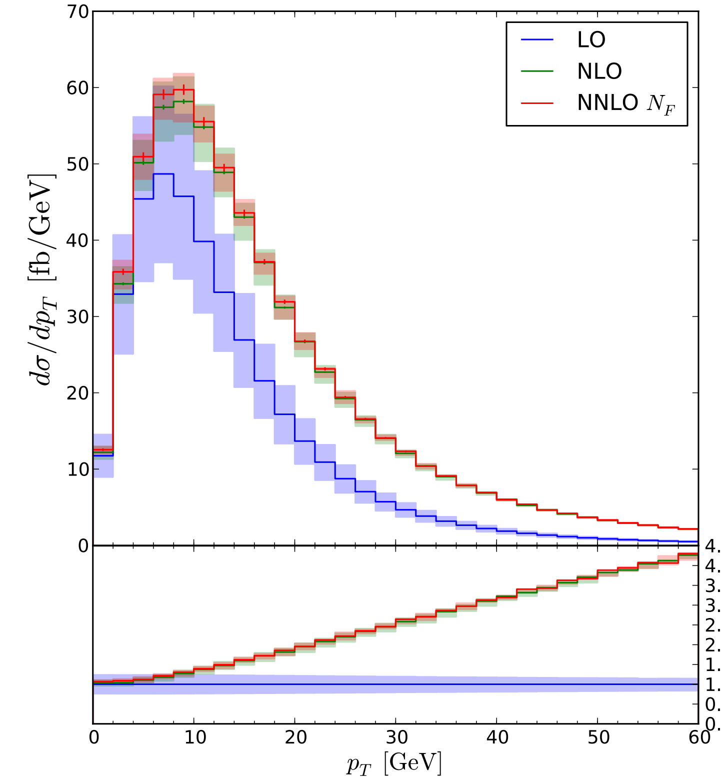
$$P^{\mu\nu} = \frac{1}{2}\left[-g^{\mu\nu} + 4k^\mu k^\nu\right] \qquad k^\mu = -\sqrt{x_3\bar{x}_3}\left[\sqrt{x_1\bar{x}_1}2\cos\pi x_4\frac{p_1^\mu - p_2^\mu}{\sqrt{s_{12}}} + (1-2x_1)\cos\pi x_4 e_1^\mu + \sin\pi x_4 e_2^\mu\right]$$

The limits are now more complicated (can be derived from Catani, Grazzini)



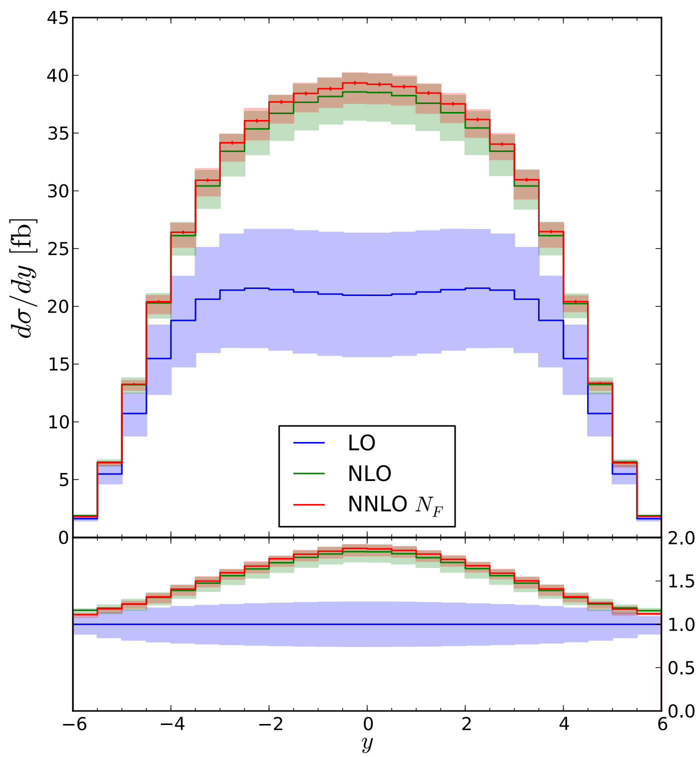

The nf corrections turn out to be very small, 1-2%. The scale uncertainty decreases, but not drastically.

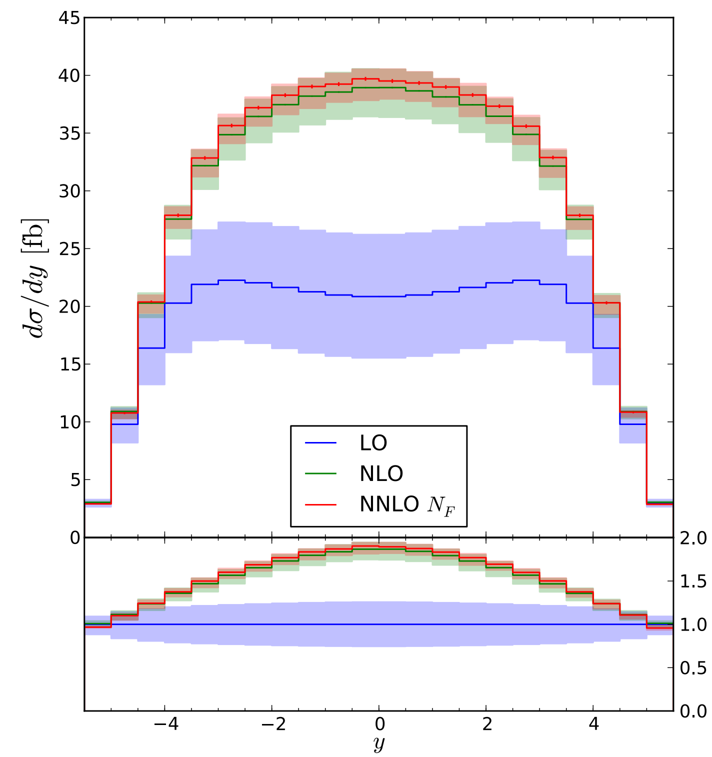



$\sqrt{s_3} = 30GeV$

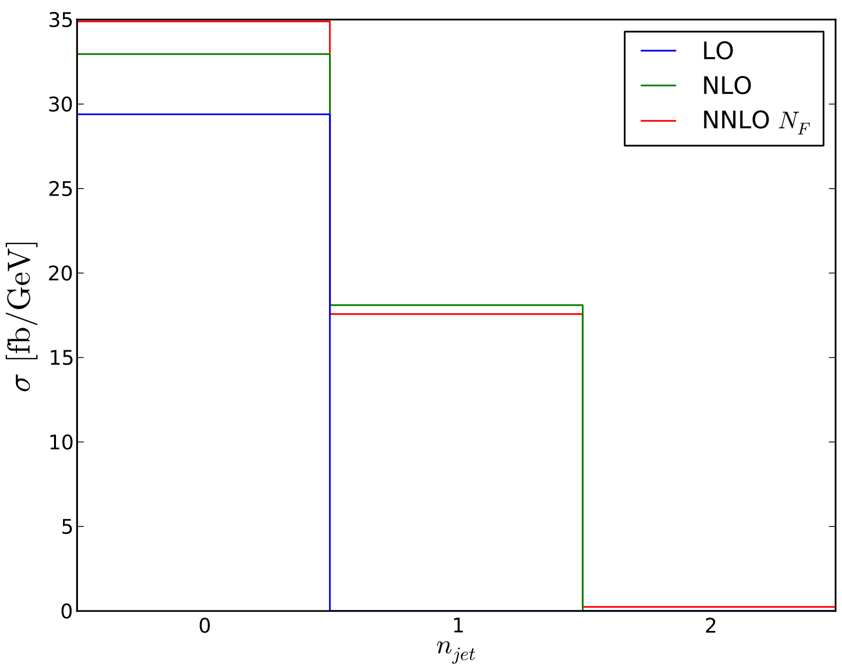
$\sqrt{s_4} = 15GeV$



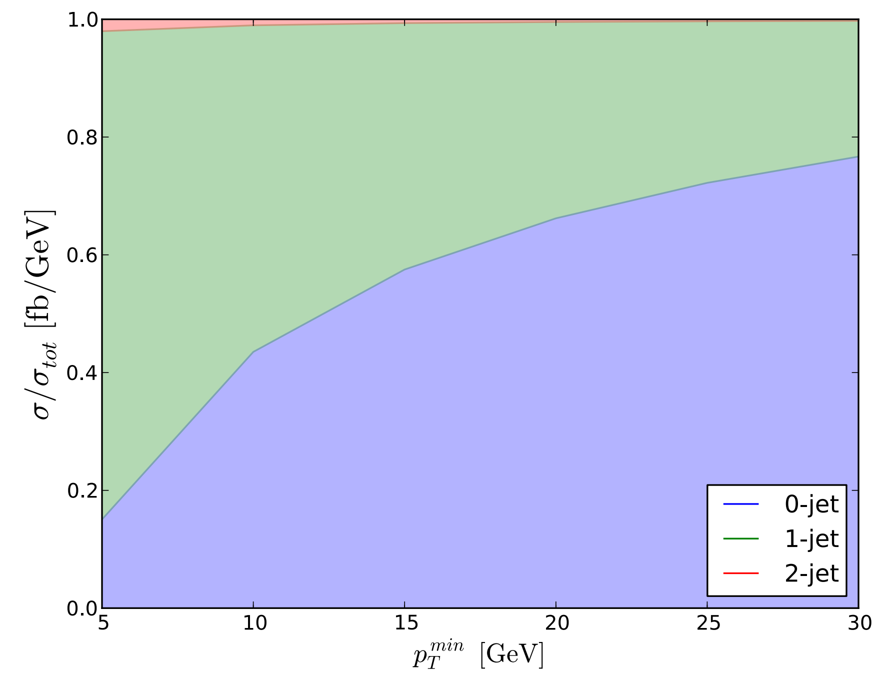




# The effect on differential distributions is small.

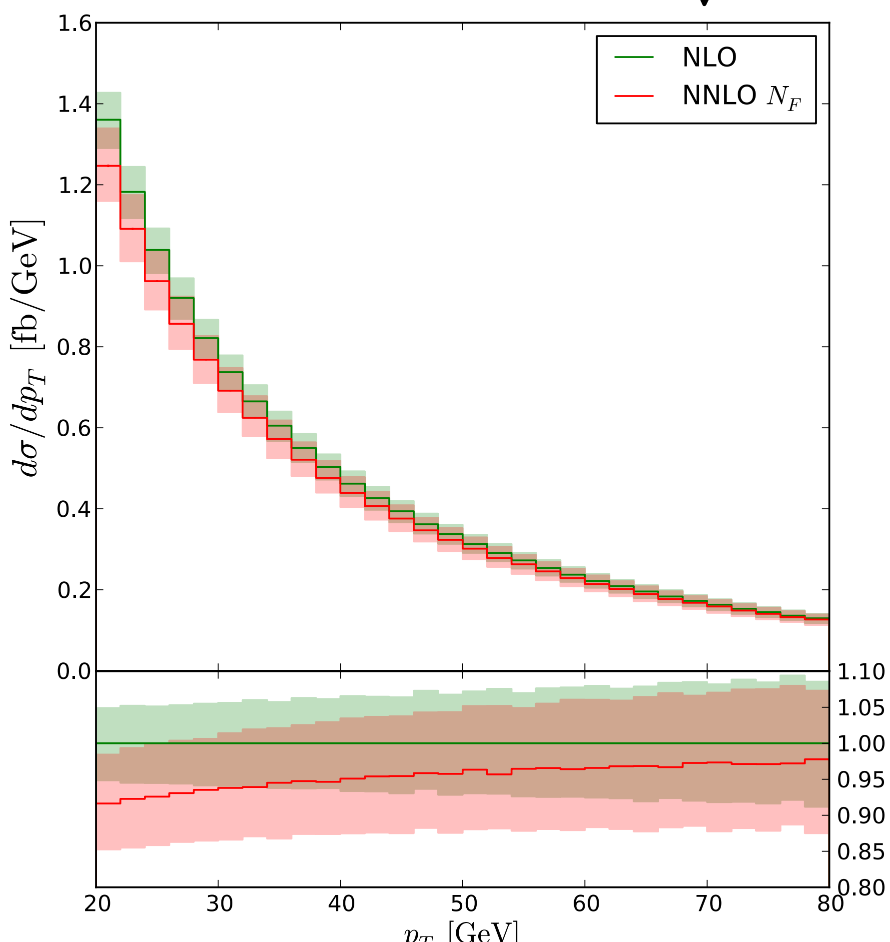
## jet cross sections for $p_T^{min} = 20GeV$

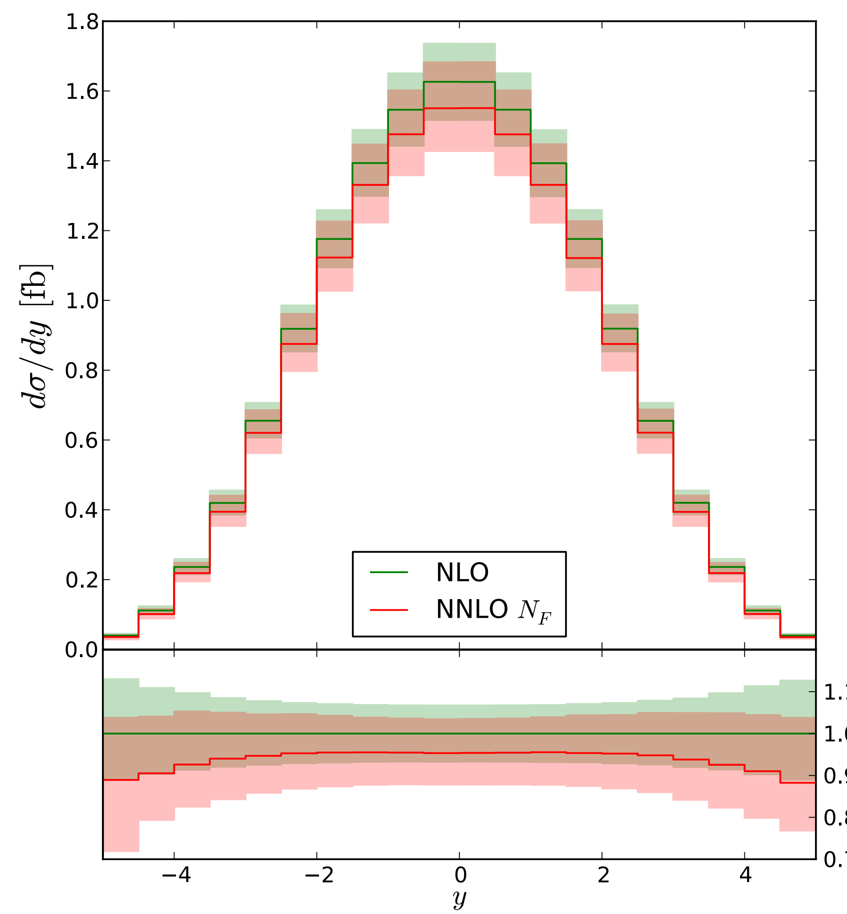



## jet cross sections as a function of $p_T^{min}$



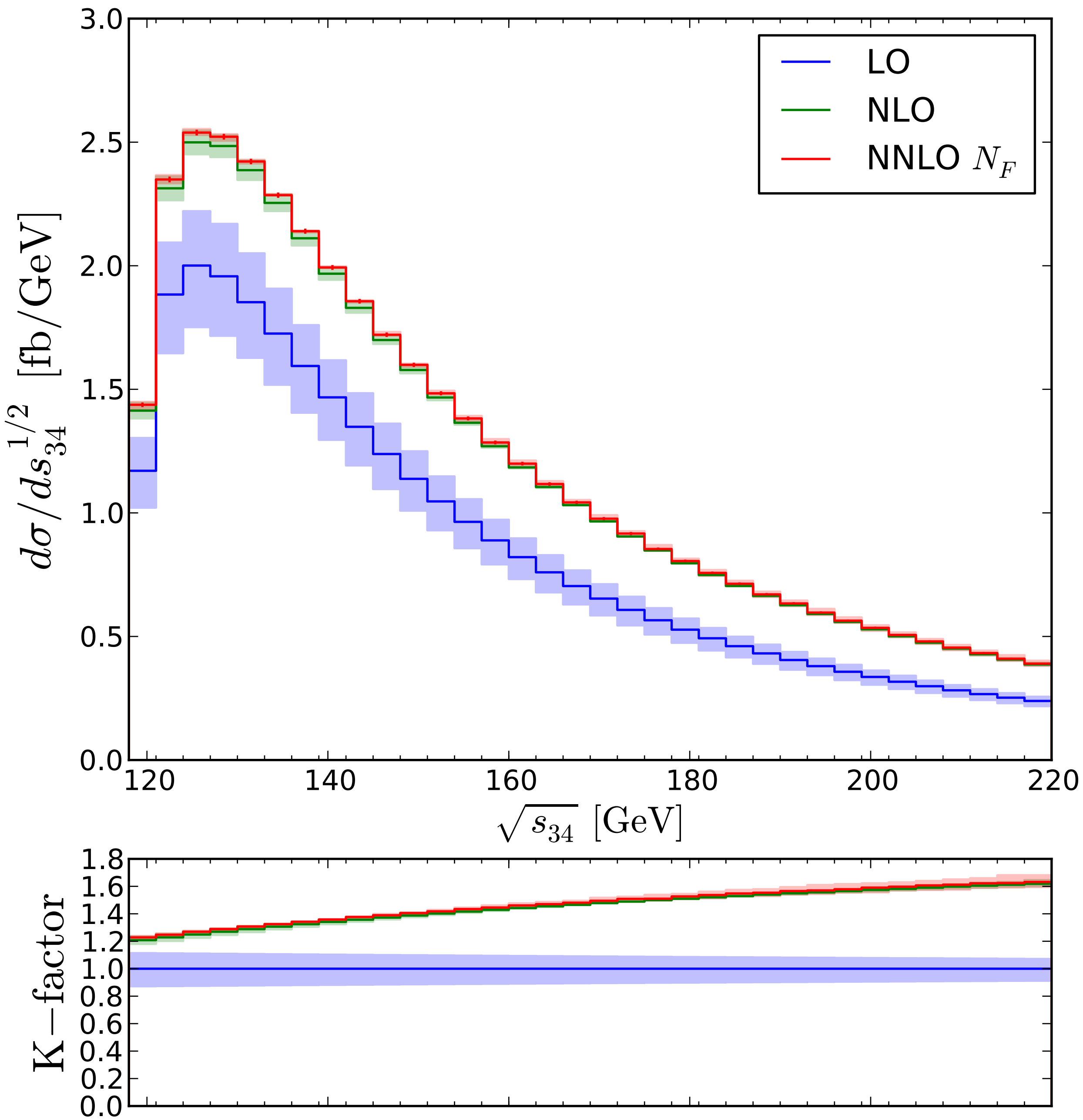

$$\sqrt{s_3} = m_Z \quad s_4 = 27GeV$$




## leading jet pT and rapidity

# invariant mass distribution of diphoton pair



The computation presented here doesn’t face the most challenging issue of overlapping singularities. It has many properties that we would like to preserve (to some extent) as we tackle the rest of the NNLO contributions: universality of counter-terms, analytic integration of counter-terms, no sector proliferation.

Thank you for your attention